Estimation of flowrate through a ruptured natural gas pipe

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Two semiempirical methods for estimating the flowrate of air through a ruptured pipe are presented. From the flowrate of air, the corresponding flowrate of methane or natural gas can then be estimated. No flowmeter is required. Details of the methods are discussed, and numerical examples are included.

Keywords: ruptured pipe; flowrate estimation; air; methane; natural gas; subsonic; sonic

Introduction

In the natural gas industry, the use of plastic pipes for underground gas reticulation is increasing. Polyethylene is the predominant material. To a much lesser extent, polyvinyl chloride and nylon-11 are also used.

The modern trend is also toward higher pressures. For domestic gas reticulation, pressures up to 500 kPa absolute are becoming increasingly common. Compared to steel, plastic pipes are more easily ruptured. With the increased pressures, the losses of valuable natural gas due to accidental ruptures usually by third parties have also increased. These gas releases are obviously not metered but would still have to be recovered from the third party involved.

For very low pressures, a method[1] of estimating this gas release is available if the pressure is less than 3 kPa gauge. Thus more versatile methods that are fast and cost-effective are required, since the cost of the estimation is not easily recoverable from the third party involved. Furthermore in most cases, a flowmeter of adequate capacity and pressure rating is not readily available. The methods must therefore not require any flowmeter. Since the form and size of the rupture can be difficult to describe, the methods must also be applicable to the actual section of pipe containing the rupture. This is the purpose of this paper.

Method

In the two semiempirical methods to be described, a common air compressor with a reservoir of known capacity is used. The section of ruptured pipe recovered from the field is connected to the reservoir at one end and capped at the other end, as shown in Figure 1. A stopwatch and a pressure gauge are also needed. It is hereby assumed that the pressure in the gas reticulation system upstream of the rupture is known. It will be shown in due course that knowing the flowrate of air through the ruptured pipe allows estimation of the corresponding flowrate of natural gas through the same pipe.

Since the composition of natural gas varies throughout the world, methane is also used here as a basis of discussion. Only a particular composition of natural gas will be considered. However, the methods are valid for other compositions of natural gas. Details of the methods are now described.

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Flowrate of air from reservoir

Consider the insulated reservoir with air escaping from a rupture, as shown in Figure 2. Let subscripts 0 and 1 denote no-flow and flow conditions, respectively, in the reservoir. Now,

$$\frac{P_0 V_0}{T_0} = \frac{P_1(V+v_1)}{T_1}$$

where P = absolute pressure
V = volume of reservoir
T = absolute temperature
v_1 = volume at pressure P_1 that escaped from reservoir

Also

$$\frac{P_\infty v_\infty}{T_\infty} = \frac{P_1 v_1}{T_1}$$

where subscript ∞ denotes ambient conditions. Therefore

$$v_\infty = \frac{T_\infty V}{P_\infty}\left[\frac{P_0}{T_0} - \frac{P_1}{T_1}\right]$$

Assuming perfect gas and adiabatic conditions throughout, we have

$$T_1/T_0 = (P_1/P_0)^{(\gamma-1)/\gamma}$$

where $\gamma = C_p/C_v$ is the ratio of specific heats. Thus

$$v_\infty = \frac{T_\infty V}{P_\infty}\left[\frac{P_0}{T_0} - \frac{1}{T_0} P_0^{(\gamma-1)/\gamma} P_1^{1/\gamma}\right]$$

$$\frac{\partial v_\infty}{\partial t} = \frac{\partial v_\infty}{\partial P_1}\frac{\partial P_1}{\partial t}$$

$$= -\frac{1}{\gamma}\frac{T_\infty V}{P_\infty T_0}\left[\frac{P_0}{P_1}\right]^{(\gamma-1)/\gamma}\frac{\partial P_1}{\partial t} \tag{1}$$

where t is the time. This is the general expression for the flowrate of air measured at ambient conditions. Depending on the value of P_1, the flow at the rupture can be sonic or subsonic. If $P_1 \geq 1.89 P_\infty$, then the flow is sonic. In the following sections, the expressions for $\partial P_1/\partial t$ will be derived for sonic and subsonic flows at the rupture.

Sonic flow

From Figure 2, the mass M_1 in the vessel is given by $M_1 = V\rho_1$, where ρ is the density. Thus

$$\frac{\partial M_1}{\partial t} = V\frac{\partial \rho_1}{\partial t} \tag{2a}$$

since V is constant. But $\partial M_1/\partial t$ can also be written as

$$\frac{\partial M_1}{\partial t} = -\rho_2 u_2 A \tag{2b}$$

where A is the rupture area and u is the velocity. The subscript 2 denotes conditions at the rupture. Using the energy equation (A.4), we have

$$V\frac{\partial \rho_1}{\partial t} = -\left\{\gamma\left[\frac{2}{\gamma+1}\right]^{(\gamma+1)/(\gamma-1)} P_1\rho_1\right\}^{1/2} A$$

Now, $P/\rho^\gamma = \text{constant} = k$, say.
This gives

$$\frac{\partial \rho_1}{\partial t} = \frac{1}{\gamma}\left[\frac{1}{k}\right]^{1/\gamma} P_1^{(1-\gamma)/\gamma}\frac{\partial P_1}{\partial t} \tag{3}$$

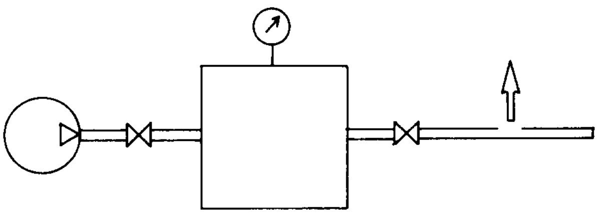

Figure 1 Schematic of experimental setup

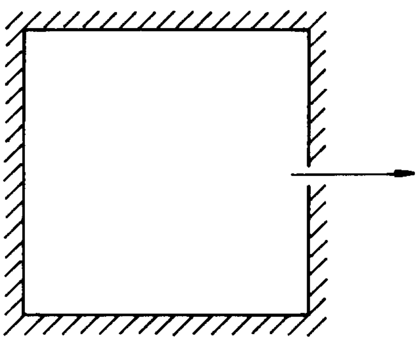

Figure 2 Flow-through rupture in insulated reservoir

Therefore

$$-V\frac{1}{\gamma}\left[\frac{1}{k}\right]^{1/\gamma} P_1^{(1-\gamma)/\gamma}\frac{\partial P_1}{\partial t} = \left\{\gamma\left[\frac{2}{\gamma+1}\right]^{(\gamma+1)/(\gamma-1)} P_1^{(\gamma+1)/\gamma}\left(\frac{1}{k}\right)^{1/\gamma}\right\}^{1/2} A$$

Rearranging, we obtain

$$-P_1^{(1-3\gamma)/(2\gamma)}\frac{\partial P_1}{\partial t} = \frac{\gamma A}{V}\left\{\gamma\left[\frac{2}{\gamma+1}\right]^{(\gamma+1)/(\gamma-1)}\left(\frac{1}{k}\right)^{1/\gamma}\right\}^{1/2} k^{1/\gamma}$$

$$= \text{constant} \qquad \text{since } A \text{ is constant}$$

$$= \theta, \quad \text{say} \tag{4}$$

Thus

$$\int_{P_0}^{P_1} P_1^{(1-3\gamma)/(2\gamma)}\,\partial P_1 = -\theta\int_0^t \partial t$$

$$\frac{2\gamma}{1-\gamma}\left[P_1^{(1-\gamma)/(2\gamma)}\right]_{P_0}^{P_1} = -\theta t$$

$$\left(\frac{P_1}{P_0}\right)^{(1-\gamma)/(2\gamma)} - 1 = \frac{\gamma-1}{2\gamma}P_0^{(\gamma-1)/(2\gamma)}\theta t$$

$$= \phi t, \quad \text{say} \tag{5}$$

where

$$\phi = \frac{\gamma-1}{2\gamma}P_0^{(\gamma-1)/(2\gamma)}\theta$$

Equation 5 shows that a plot of $(P_1/P_0)^{(1-\gamma)/(2\gamma)} - 1$ against t should give a straight line with slope ϕ through the origin. This value of ϕ needs to be determined experimentally. From Equation 4,

$$\frac{\partial P_1}{\partial t} = -\phi\frac{2\gamma}{\gamma-1}P_0^{(1-\gamma)/(2\gamma)}P_1^{(3\gamma-1)/(2\gamma)} \tag{6}$$

Substitution of Equation 6 into Equation 1 gives the required flowrate for sonic conditions at the rupture.

Subsonic flow

Referring again to Figure 2, we hereby assumed that $P_2 = P_\infty$. From Equation 2 and the energy equation (A.1), we have

$$V\frac{\partial \rho_1}{\partial t} = -\left\{\frac{2\gamma}{\gamma-1}P_1\rho_1\left[\left(\frac{P_2}{P_1}\right)^{2/\gamma} - \left(\frac{P_2}{P_1}\right)^{(\gamma+1)/\gamma}\right]\right\}^{1/2}$$

Notation

A	Area of rupture
C_p	Specific heat at constant pressure
C_v	Specific heat at constant volume
g	Acceleration due to gravity
I	An integral
k	Constant $= P/\rho^\gamma$
M	Mass
P	Absolute pressure
P_w	Operating absolute pressure of pipe
Q	Volume flowrate
R	Specific gas constant
T	Absolute temperature
t	Time
u	Flow velocity
V	Volume of air reservoir
v	Volume of air that escaped
w	$(\alpha x^2 - \beta)^{1/2}$
x	$P_1^{(\gamma-1)/2\gamma}$
z	Height above an arbitrary datum
α	$P_2^{2/\gamma}$
β	$P_2^{(\gamma+1)/\gamma}$
γ	Ratio of specific heats $= C_p/C_v$
ζ	Scaling factor
θ	Constant
λ	Constant
ξ	Slope
ρ	Density
ϕ	Slope

Subscripts

0	Conditions in reservoir at no-flow
1	Conditions in reservoir at flow
2	Conditions at rupture
∞	Ambient conditions

Letting $P/\rho^{\gamma}=k$ as before and using Equation 3 gives

$$-\frac{V}{\gamma}\left(\frac{1}{k}\right)^{1/\gamma}P_1^{(1-\gamma)/\gamma}\frac{\partial P_1}{\partial t}$$
$$=\left\{\frac{2\gamma}{\gamma-1}P_1^{(\gamma+1)/\gamma}\left(\frac{1}{k}\right)^{1/\gamma}\left[\left(\frac{P_2}{P_1}\right)^{2/\gamma}-\left(\frac{P_2}{P_1}\right)^{(\gamma+1)/\gamma}\right]\right\}^{1/2}A$$

Rearranging gives

$$-\frac{P_1^{(1-3\gamma)/(2\gamma)}}{[(P_2/P_1)^{2/\gamma}-(P_2/P_1)^{(\gamma+1)/\gamma}]^{1/2}}\frac{\partial P_1}{\partial t}=\frac{\gamma A}{V}k^{1/\gamma}\left[\frac{2\gamma}{\gamma-1}\left(\frac{1}{k}\right)^{1/\gamma}\right]^{1/2}$$

$=\text{constant}$

since A is constant

$=\lambda$, say

Let $\alpha=P_2^{2/\gamma}$ and $\beta=P_2^{(\gamma+1)/\gamma}$.

Thus

$$\lambda=-\frac{P_1^{(1-3\gamma)/(2\gamma)}}{[\alpha P_1^{-2/\gamma}-\beta P_1^{-(\gamma+1)/\gamma}]^{1/2}}\frac{\partial P_1}{\partial t}$$
$$=-\frac{1}{[\alpha P_1^{3(\gamma-1)/\gamma}-\beta P_1^{2(\gamma-1)/\gamma}]^{1/2}}\frac{\partial P_1}{\partial t} \quad (7)$$

$$\int_0^t \lambda\,\partial t=-\int_{P_0}^{P_1}\frac{1}{[\alpha P_1^{3(\gamma-1)/\gamma}-\beta P_1^{2(\gamma-1)/\gamma}]^{1/2}}\partial P_1$$
$$\lambda t=-[I(P_1)]_{P_0}^{P_1}$$
$$=I(P_0)-I(P_1) \quad (8)$$

where

$$I(P_1)=\int\frac{1}{[\alpha P_1^{3(\gamma-1)/\gamma}-\beta P_1^{2(\gamma-1)/\gamma}]}\partial P_1$$

Here, $I(P_1)$ is taken to denote the integral I as a function of P_1. Substituting P_0 in place of P_1 gives $I(P_0)$.

Let $x=P_1^{(\gamma-1)/(2\gamma)}$. This gives

$$\partial P_1=\frac{2\gamma}{\gamma-1}x^{(\gamma+1)/(\gamma-1)}\partial x$$

Therefore

$$I(x)=\frac{2\gamma}{\gamma-1}\int\frac{x^{(\gamma+1)/(\gamma-1)}}{[\alpha x^6-\beta x^4]^{1/2}}\partial x$$

For air, it is assumed that $\gamma=1.4$. Thus

$$I(x)=\frac{2\gamma}{\gamma-1}\int\frac{x^4}{[\alpha x^2-\beta]^{1/2}}\partial x$$

The solution[2] is

$$I(P_1)=\frac{2\gamma}{\gamma-1}\left[\frac{1}{4}\frac{x^3w}{\alpha}+\frac{3}{8}\frac{\beta xw}{\alpha^2}+\frac{3}{8}\frac{\beta^2}{\alpha^{5/2}}\ln(\alpha^{1/2}x+w)\right] \quad \text{for } \alpha>0 \quad (9)$$

where $x=P_1^{(\gamma-1)/(2\gamma)}$
$\alpha=P_2^{2\gamma}$
$\beta=P_2^{(\gamma+1)/\gamma}$
$w=(\alpha x^2-\beta)^{1/2}$

From Equation 8

$$1-\frac{I(P_1)}{I(P_0)}=\frac{\lambda}{I(P_0)}t=\xi t \quad (10)$$

where $\xi=\lambda/I(P_0)$. Equation 10 shows that a plot of $1-I(P_1)/I(P_0)$ against t should give a straight line with slope ξ through the origin. The value of ξ needs to be determined experimentally. From Equation 7,

$$\frac{\partial P_1}{\partial t}=-\xi I(P_0)[\alpha P_1^{3(\gamma-1)/\gamma}-\beta P_1^{2(\gamma-1)/\gamma}]^{1/2} \quad (11)$$

Substitution of Equation 11 into Equation 1 gives the required flowrate for subsonic conditions at the rupture. Note that, by assumption, $P_2=P_\infty$. Also the expression for $I(P_1)$ as given by Equation 9 only holds for $\gamma=1.4$.

Experimental results

With the setup shown in Figure 1, value V1 is shut off when the pressure in the reservoir has reached the required value P_0. A value of $P_0\simeq P_w$ of the ruptured pipe is preferable. Furthermore, the reservoir is then allowed to reach equilibrium with ambient temperature. This gives a convenient value of $T_0\simeq T_\infty$.

Valve V2 should be a full-bore ball valve for fast action and minimum pressure drop. The capacity of the reservoir used here is $V=0.093\,m^3$. The pipe inside diameter is 26.5 mm.

To start the test, valve V2 is opened to allow air to escape through the rupture. The pressure P_1 in the reservoir and the time elapsed t are recorded.

For sonic flows, typical plots of $(P_1/P_0)^{(1-\gamma)/(2\gamma)}-1$ against t for a particular rupture area are shown as S1 in Figure 3. Except when $P_1\rightarrow 1.89P_\infty$ with increasing t, it can be seen that agreement with Equation 5 is good. When the rupture is enlarged, the resulting plots are shown as S2 in Figure 3. Compared to S1, agreement of S2 with Equation 5 is improved. As $P_1\rightarrow 1.89P_\infty$ with increasing t, S1 and S2 begin to deviate from Equation 5. Maximum P_0 used is 1100 kPa.

For $P_0<1.89P_\infty$, the flow at the rupture is subsonic. Typical plots of $1-I(P_1)/I(P_0)$ against t are shown as S3 in Figure 4. Except when $P_1\rightarrow P_\infty$ with increasing t, it can be seen that agreement with Equation 10 is good. As $P_1\rightarrow P_\infty$ with increasing t, S3 begins to deviate from Equation 10 in a manner similar to S1 and S2.

The deviations in Figures 3 and 4 emphasize the need to keep P_0 as close as possible to P_w. This is also when the assumption of an adiabatic condition is most realistic.

An interesting feature of every plot is that the slope is independent of P_0. This points to the possibility of limited extrapolation when $P_w>P_0$.

Only $\partial v_\infty/\partial t$ remains to be calculated. This is dealt with next.

Sample calculations

Given below are numerical examples to illustrate the use of the two methods for estimating the flowrate of air. When $P_w\geq 1.89P_\infty$, the sonic method applies. When $P_w<1.89P_\infty$, the subsonic method applies instead. Once the flowrate of air is known, the corresponding flowrate of methane or natural gas can be estimated. For convenience, it is assumed that $T_0=T_\infty=288.15$ K and $P_\infty=101.325$ kPa. Note that $V=0.093\,m^3$.

Sonic method

Consider Figure 5, which is obtained from a polyethylene pipe with an inside diameter of 26.5 mm. The pipe was deformed and ruptured by a third-party excavator.

(a) $P_0=308$ kPa, $\phi=0.006\,s^{-1}$ (experimentally determined).
(b) Take $P_w=308$ kPa.
(c) Putting P_w in place of P_1 in Equation 6 gives

$$\partial P_w/\partial t=-12{,}936\text{ Pa/s}$$

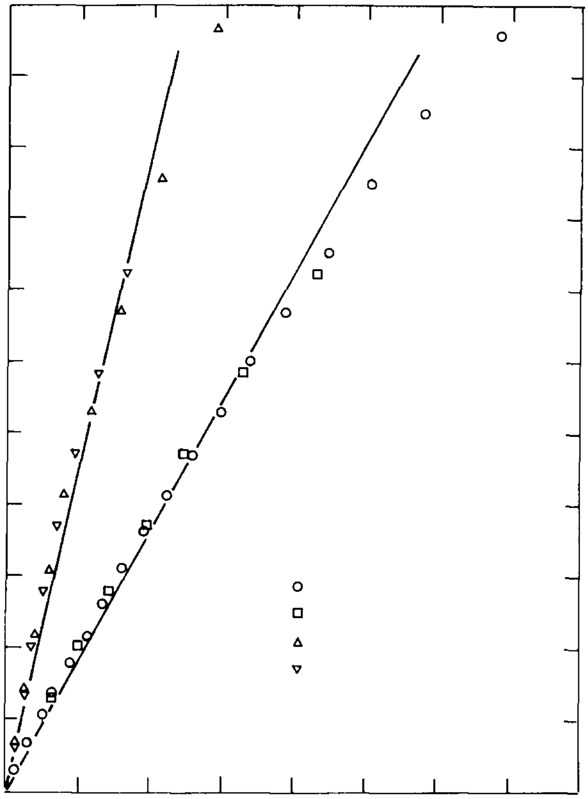

Figure 3 Sonic flow of air

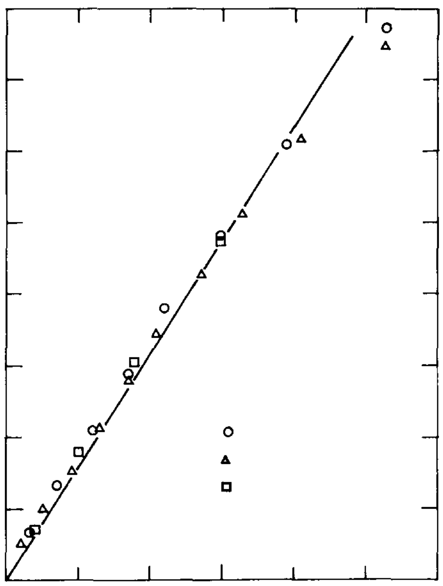

Figure 4 Subsonic flow of air

(d) Putting P_w in place of P_1 in Equation 1 gives

$$\partial v_\infty/\partial t = 8.481 \times 10^{-3}\,m^3/s = 30.53\,m^3/h$$

(e) Putting P_w in place of P_1 in Figures A2 and A3, we have $\zeta = 1.309$ for methane and 1.171 for natural gas.

(f) Flowrate of methane = $39.96\,m^3/h$.
Flowrate of natural gas = $35.75\,m^3/h$.

It is noteworthy in this case that, because of the rather rapid depressurization of the reservoir, it might have been better to increase V or P_0 or both. It is also preferable to use a pressure recorder if possible.

Subsonic method

Consider S3 in Figure 4. Note that, by assumption, $P_2 = P_\infty$.

(a) $P_0 = 191$ kPa.
From Equation 9, $I(P_0) = 5.661$ in consistent units.

$$\xi = 1.58 \times 10^{-3}\,s^{-1} \qquad \text{(experimentally determined)}$$

(b) Take $P_w = 151$ kPa.
$I(P_w) = 5.497$ in consistent units.

(c) Putting P_w in place of P_1 in Equation 11 gives

$$\partial P_w/\partial t = -1838\,Pa/s$$

(d) Putting P_w in place of P_1 in Equation 1 gives

$$\partial v_\infty/\partial t = 1.289 \times 10^{-3}\,m^3/s = 4.638\,m^3/h$$

(e) Putting P_w in place of P_1 in Figures A2 and A3, we have $\zeta = 1.321$ for methane and 1.184 for natural gas.

(f) Flowrate of methane = $6.13\,m^3/h$.
Flowrate of natural gas = $5.49\,m^3/h$.

Discussion

As a check on the estimated flowrate of air, the ruptured pipe from Figure 5 was connected to a compressed air pipeline as shown in Figure 6. A standard positive-displacement diaphragm-type gas flowmeter (American Meter Company AL5000) was

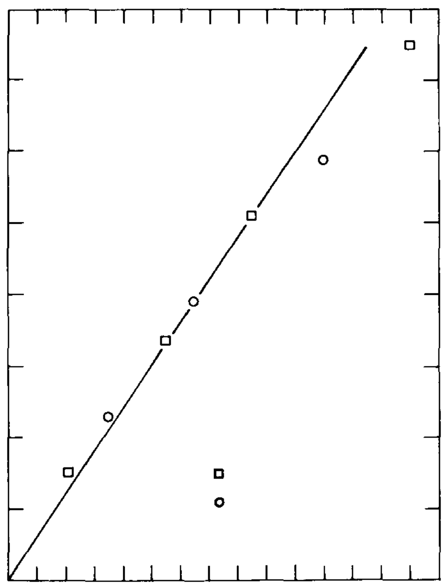

Figure 5 Sonic flow of air through ruptured polyethylene pipe

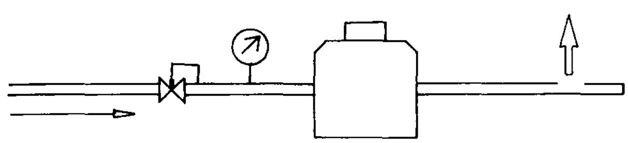

Figure 6 Measurement of the actual flowrate using a flowmeter

used to measure the flowrate. At flow conditions, the static pressure in the pipeline as indicated by the pressure gauge was set to 308 kPa. The measured flowrate of air was 32.8 m^3/h. Thus the estimated flowrate of 30.53 m^3/h was lower than the measured flowrate by about 7%, which is quite acceptable. The lack of a gas-flaring facility prevented a check on the estimated flowrate of natural gas.

A reason for the discrepancy is the way the pressures were taken. In Figure 1, the pressure gauge indicates the stagnation pressure P_1. In Figure 6, the pressure gauge indicates the static pressure. The stagnation pressure in Figure 6 is thus higher than P_1. Had a stagnation pressure probe instead of a simple wall tapping been used in Figure 6, the discrepancy in the flowrates would be smaller.

The initial dynamically changing situation in Figure 1 takes place very rapidly and is not expected to have a major effect on the estimated flowrate. If the rupture is very large, Figure 1 will not be suitable unless the reservoir is also very large because the depressurization will be too rapid for accurate measurements of the instantaneous reservoir pressure.

As mentioned earlier, the pressure profile upstream of the rupture is assumed to be known. Since the demand for gas is likely to change throughout the day, the upstream pressure is also likely to change. If the profile is not known, it may be obtained by using specialized computer programs. An example is *STAG*,[3] which is a program for the steady-state and transient analysis of gas reticulation systems.

Conclusions

(a) The setup as shown in Figure 1 is adequate for estimating the flowrate of air through a ruptured pipe. A reservoir of adequate capacity is needed. No flowmeter is required.
(b) Flowrates estimated by using Figure 1 will be lower than those measured by using Figure 6 because the pressure gauge in Figure 1 indicates the stagnation pressure, whereas the pressure gauge in Figure 6 indicates the static pressure.
(c) Two methods are described: one for sonic flows and the other for subsonic flows. The operating pressure P_w of the ruptured pipe dictates the method to be used. The resulting flowrate of air is then scaled to obtain the corresponding flowrates of methane and natural gas.
(d) For a given ruptured pipe, the slope ϕ of plots of $(P_1/P_0)^{(1-\gamma)/(2\gamma)}-1$ against time and slope ξ of plots of $1-I(P_1)/I(P_0)$ against time were found experimentally to be independent of the reservoir pressure P_0. This allows limited extrapolation when $P_w > P_0$. In the experiments, maximum $P_0 = 1100$ kPa.
(e) It is preferable to keep P_0 as close as possible to P_w.
(f) The methods are fast and cost-effective.

Acknowledgments

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Appendix: Scaling factors

For flows with appreciable changes of density, the energy equation[4] for a steady adiabatic flow of a perfect gas is

$$\frac{\gamma}{\gamma-1}\frac{P}{\rho}+\tfrac{1}{2}u^2+gz=\text{constant}$$

where g is the acceleration due to gravity and z is the height above an arbitrary datum. For gases, the term gz is negligible. Referring to Figure 2, we get

$$\frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1}=\frac{\gamma}{\gamma-1}\frac{P_2}{\rho_2}+\tfrac{1}{2}u_2^2$$

$$u_2^2=\frac{2\gamma}{\gamma-1}\left[\frac{P_1}{\rho_1}-\frac{P_2}{\rho_2}\right]$$

$$\rho_2^2u_2^2=\frac{2\gamma}{\gamma-1}\frac{P_1}{\rho_1}\rho_1^2\left[\frac{\rho_2^2}{\rho_1^2}-\frac{P_2}{P_1}\frac{\rho_2}{\rho_1}\right]$$

The mass flowrate is

$$\rho_2u_2A=\left\{\frac{2\gamma}{\gamma-1}P_1\rho_1\left[\left(\frac{P_2}{P_1}\right)^{2/\gamma}-\left(\frac{P_2}{P_1}\right)^{(\gamma+1)/\gamma}\right]\right\}^{1/2}A \qquad \text{(A1)}$$

$$=\left\{\frac{2\gamma}{\gamma-1}\frac{P_1^2}{RT_1}\left[\left(\frac{P_2}{P_1}\right)^{2/\gamma}-\left(\frac{P_2}{P_1}\right)^{(\gamma+1)/\gamma}\right]\right\}^{1/2}A \qquad \text{(A2)}$$

since $P_1/\rho_1^\gamma=P_2/\rho_2^\gamma$ and $P=\rho RT$ where R is the specific gas constant.

For maximum ρ_2u_2A, we require

$$\frac{\partial(\rho_2u_2A)}{\partial(P_2/P_1)}=0$$

or

$$\frac{P_2}{P_1}=\left[\frac{2}{c+1}\right]^{\gamma/(\gamma-1)}$$

Thus sonic flow occurs at the rupture when

$$\frac{P_2}{P_1}\geq\left[\frac{\gamma+1}{2}\right]^{\gamma/(\gamma-1)} \qquad \text{(A3)}$$

$$=\begin{cases}1.893 & \text{when } \gamma=1.4 \text{ for air}\\ 1.832 & \text{when } \gamma=1.3 \text{ for methane}\end{cases}$$

At sonic flow, Equation A1 becomes

$$\rho_2u_2A=\left\{\gamma\left[\frac{2}{\gamma+1}\right]^{(\gamma+1)/(\gamma-1)}P_1\rho_1\right\}^{1/2}A \qquad \text{(A4)}$$

$$=\left\{\gamma\left[\frac{2}{\gamma+1}\right]^{(\gamma+1)/(\gamma-1)}\frac{P_1^2}{RT_1}\right\}^{1/2}A \qquad \text{(A5)}$$

At ambient conditions, the volume flowrate

$$Q_\infty=\rho_2u_2A/\rho_\infty$$

Equation (A5) shows that for a given T_1 and A,

$$Q_\infty/A=\rho_2u_2/\rho_\infty=\text{constant}\times P_1$$

That is, plots of Q_∞/A against P_1 are straight lines through the origin, as shown in Figure A1. The values[5] of R used are 286.8 J(kg K) and 518 J/(kg K) for air and methane, respectively. Also, $T_1 = 288.15$ K, $\rho_{air} = 1.225$ kg/m^3, and relative density of methane is 0.554.

The ratio of the flowrate of methane to the flowrate of air is shown in Figure A2. This gives the scaling factor ζ required

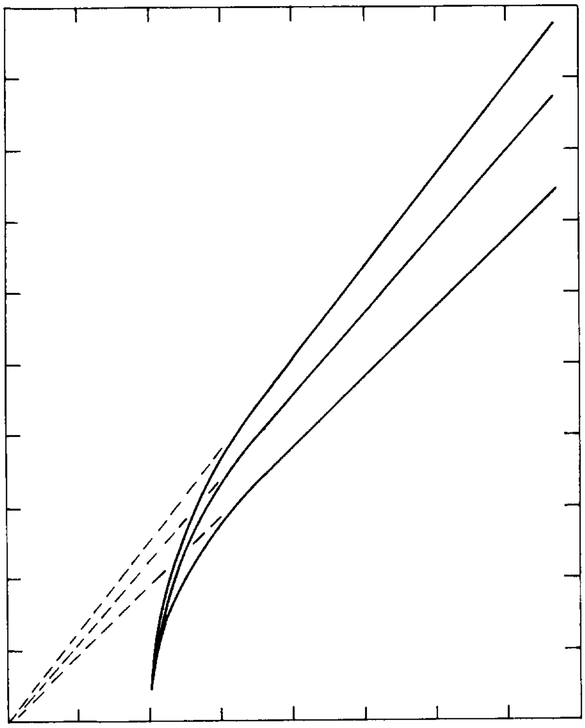

Figure A1 Comparative flowrates for different gases: P_∞ = 101.325 kPa, T_∞ = 288.15 K

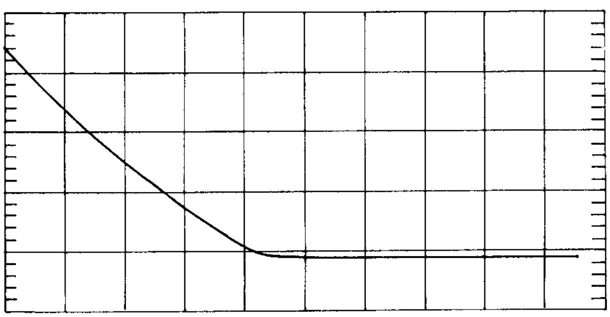

Figure A2 Scaling factor ζ for methane: P_∞ = 101.325 kPa, T_∞ = 288.15 K

Table 1 Typical percentage mole fractions of natural gas available in Auckland

Methane	82.40
Ethane	7.07
Propane	2.61
I-butane	0.44
N-butane	0.44
I-pentane	0.10
N-pentane	0.06
Hexane+	0.04
Carbon dioxide	4.72
Nitrogen	2.12
Total	100.00

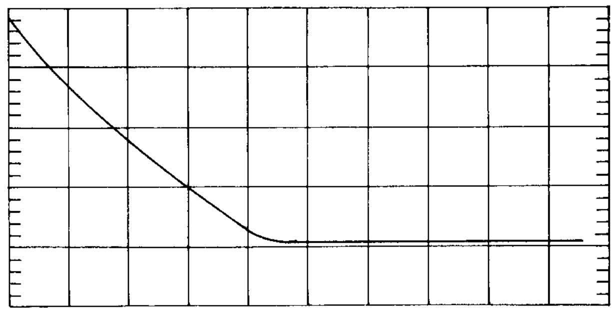

Figure A3 Scaling factor ζ for natural gas: P_∞ = 101.325 kPa, T_∞ = 288.15 K

when estimated flowrates of air are converted to flowrates of methane.

When flowrates of natural gas are required, then the appropriate values of γ and R need to be known. Table 1 shows a typical composition of natural gas available in Auckland. Using the data and the weighted-value approach adopted in the *GPSA Engineering Data Book*,[6] we obtain

Molecular weight = 19.829

$C_p = 1923$ J/(kg K)

$C_v = 1504$ J/(kg K)

$\gamma = 1.28$

$R = 419$ J/(kg K)

$$\text{Relative density} = \frac{\text{Mol. wt. of natural gas}}{\text{Mol. wt. of air}} = 0.6846$$

The relative density may also be obtained by using ISO 6976.[7]

The plots of Q_∞/A and ζ against P_1 for natural gas are shown in Figures A1 and A3, respectively. Extension to other gases or mixture of gases is quite straightforward.